Complex Demodulation

1. The data *X(t)* is taken to be a nearly-periodic signal plus everything else, *Z(t)*. The amplitude *A* and phase φ of the periodic signal are allowed to be time-dependent but assumed to vary slowly compared to the frequency ω.

$$X(t) = A(t)\cos(\omega t + \varphi(t)) + Z(t)$$
$$= \frac{A(t)}{2}\left[e^{i(\omega t + \varphi(t))} - e^{-i(\omega t + \varphi(t))}\right] + Z(t)$$

2. "Demodulate" by multiplying by $e^{-i\omega t} \quad \rightarrow \quad Y(t) = X(t)e^{-i\omega t}$, which can be written:

$$Y(t) = \underset{(a)}{\frac{A(t)}{2}e^{i\varphi(t)}} + \underset{(b)}{\frac{A(t)}{2}e^{-i(2\omega t+\varphi(t))}} + \underset{(c)}{Z(t)e^{-i\omega t}}$$

Term (a) varies slowly, with no power at or above frequency ω;
Term (b) varies at frequency 2ω;
Term (c) varies at frequency ω. (Note that by postulate *Z(t)* has no power at frequency ω, so the shifted term (c) has no power at zero frequency.)

3. Low-pass filter to remove frequencies at or above frequency ω.
This (nearly) removes terms (b) and (c), and smooths (a). The result is:

$$Y'(t) \approx \frac{A'(t)}{2}e^{i\varphi'(t)},$$ where the prime indicates smoothing.

The choice of smoother determines the width of the frequency band retained. For triangle smoothing with length (2T – 1), where T is the demodulation period, T = 2π/ω, the half-power bandwidth is from T/(1+0.44295) to T/(1–0.44295), or from about 0.69T to 1.8T.

4. Extract A' and φ': $A'(t) = 2|Y'| = 2\left(\mathrm{Re}\{Y'\}^2 + \mathrm{Im}\{Y'\}^2\right)^{1/2}, \quad \varphi'(t) = \mathrm{atan}\left(\frac{\mathrm{Im}\{Y'\}}{\mathrm{Re}\{Y'\}}\right)$

5. The choice of ω can be checked by locally fitting a line to the phase: $\varphi(t) \approx \gamma t + b$.
Typically this is done over running intervals of length T. With the origin chosen at the central time of each interval (so $b \approx 0$), this gives cos(ωt+φ) ≈ cos(ωt+γt) = cos(ω^*t).
The fitted frequency ω^*(t) = ω + γ is a check on the initial choice ω.

Reference: Bloomfield, P. (1976). Fourier Decomposition of Time Series, An Introduction. Wiley, New York, 258pp.